Dynamic Hedging in Stock Index Futures via Copula Multiplicative Error Model

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ABSTRACT

This paper combines a copula function and multiplicative error models to capture the dependence structure and the volatility patterns simultaneously, named cMEM. We examine hedging performance of the presenting cMEM with different estimation window sizes for the futures contract of Taiwan stock price index. The results have shown that the cMEM with 1250-day window size for Clayton survival, Gumbel and OLS has better performance in which Clayton survival survives during the crisis and has the best out-of-sample hedging effectiveness. The empirical evidence indicates that the cMEM performs well for the turmoil periods.

Keywords: hedge ratio; copula; multiplicative error model; dependence structure; stock futures

1. Introduction

Accurate forecasts for volatility of and correlation between spot and futures are important for measuring minimum-variance hedge ratios (MVHR). To do that, previous literature often uses bivariate GARCH-type models (Baillie and Myers, 1991; Kroner and Sultan, 1993; Park and Switzer, 1995; Gagnon and Lypny, 1995; Kavussanos and Nomikos, 2000; Bystrom, 2003; Lee and Yoder, 2007). However, Longin and Solnik (2001) and Ang and Chen (2002), among others, have provided evidence of the asymmetric dependence between stock returns, indicating the conventional assumption of joint normality or joint elliptical distribution has become inadequate, which is the key assumption for bivariate GARCH-type models. Recently, more and more studies deal with the dependence structure in estimation of optimal hedge ratios. Dependence structure is a broader concept of market co-movement than implied by linear correlation such as the tail dependence and the nonlinear and/or asymmetric dependence implied by the shape of the joint distribution. Copula-based model has proven performed well in capturing the patterns and forecasting dependence structure. The copula method has several merits. First, the copula functions allow non-linear dependence structure, and some of them exhibit asymmetric dependence and tail dependence between spot and futures returns which is closer to the reality that spot and futures markets boom together or collapse together more often than that implied by joint normality. Second, copula method enables to deal with the specification of marginal distributions separately from the specification of market co-movement and dependence. That means the marginal distributions and the joint distribution implied by copula functions need not necessarily belong to the same family, providing flexibility in modelling the joint distributions and easing the computational efforts in estimation (Joe and Xu, 1996). Lai, Chen and Gerlach (2008), Lai (2009) and Lee (2009) apply various dependence structures implied by copula functions to estimating time-varying minimum-variance hedge ratios and find that the model can improve hedging performance in terms of variance reduction.

Most of the papers using copula approach still employ the GARCH-type model as marginal model. However, multiplicative error model (MEM) has recently been applied widely to the non-negative random variables such as volatility, volume and number of trades. MEM has also been proven performed better in forecasting volatility than the GARCH-type models (Engle and Gallo, 2006; Cipollini, Engle and Gallo, 2009; Brownlees and Gallo, 2009). To exploit the advantages of copula and multiplicative error models, this paper combines a copula function and two univariate MEMs proposed by Brownlees and Gallo (2009) to build the joint distribution of spot and futures returns and develop a copula-MEM (cMEM) framework for dynamic minimum-variance hedging in stock futures of Taiwan. The results show that the best cMEM outperforms the conventional approaches such as the ordinary least square (OLS) and the error correction model (ECM) by more than 11% for out-of-sample during 2007-2010 including the sub-prime crisis.

In addition, this paper compares different strategies with different estimation windows. Since MVHR is calculated based on the latest available information set, the size of the estimation window for dependence structure may play an important role in out-of-sample rolling hedging. Too larger window could dilute the impact of the latest observations and can not reflect the change of the dependence structure immediately, possibly resulting in poor hedging performance. However, too small window could incur estimation errors for dependence structure. This paper investigates five different sizes (500/750/1000/1250/1500) of estimation window and evaluates the hedging performance. The results show that cMEMs with 1250 estimation window provide better hedging performance.

The outline of this article is as follows. In the next section, the copula functions, the MEMs and the copula MEM are introduced. In the following section, the hedging performance criterion is discussed. Data descriptions and empirical results are reported and discussed in the fourth section, and the last section concludes.

2. Methodology

2.1 Univariate multiplicative model (MEM) for volatility

The MEM of observed volatility (standard deviation) $V_{i,t}$ for spot returns $r_{1,t}$ and futures returns $r_{2,t}$ is assumed

$$V_{i,t}^2 = \sigma_{i,t}^2 \varepsilon_{i,t} \tag{1}$$

where $\varepsilon_{i,t} \mid I_{t-1} \sim Gamma(\phi_i, \phi_i)$; ϕ_i is the shape parameter; I_{t-1} is the information set at $t-1$; $E(\varepsilon_{i,t} \mid I_{t-1}) = 1$ and $V(\varepsilon_{i,t}) = 1/\phi_i$. Consequently, $V_{i,t}^2 \mid I_{t-1} \sim Gamma(\phi_i, \phi_i / \sigma_{i,t}^2)$, $E(V_{i,t}^2 \mid I_{t-1}) = \sigma_{i,t}^2$ and $V(V_{i,t}^2 \mid I_{t-1}) = \sigma_{i,t}^2 / \phi_i$. $\sigma_{i,t}^2$ is a nonnegative conditionally predictable process with asymmetric response for the past daytime(open-to-close) returns $r_{i,t-1}^{OC}$, specified as

$$\sigma_{i,t} = \alpha + \beta V_{i,t-1} + \gamma \sigma_{i,t-1} + \beta^- V_{i,t-1} I(r_{t-1}^{OC} < 0) \tag{2}$$

where $I(\cdot)$ is an indicator function, and $I(r_{i,t-1}^{OC} < 0) = 1$ if daytime returns $r_{i,t-1}^{OC} < 0$ and 0 otherwise. Since $V_{i,t}^2 \mid I_{t-1} \sim Gamma(\phi_i, \phi_i / \sigma_{i,t}^2)$, the log-likelihood function can be formulated as

$$l_t = \ln L_t = \phi_i \ln \phi_i - \ln \Gamma(\phi_i) + (\phi_i - 1) \ln V_{i,t}^2 - \phi_i (\ln \sigma_{i,t}^2 + V_{i,t}^2 / \sigma_{i,t}^2) \tag{3}$$

All estimates are obtained by the maximum likelihood estimation (MLE).

2.2 Copula function, marginal distributions of spot and futures returns and cMEM

A copula is a multivariate cumulative distribution function whose marginal distribution is uniform on the interval [0,1]. It captures the dependence structure of a multivariate distribution. According to Sklar's (1959) theorem, a bivariate joint cumulative distribution function (F) of spot returns $r_{1,t}$ and futures returns $r_{2,t}$ can be decomposed into two marginal cumulative distribution functions (F_1 and F_2) and a copula cumulative distribution function (C) that completely describes the dependence structure between the two series:

$$F(r_{1,t}, r_{2,t}; \theta_1, \theta_2, \rho) = C(F_1(r_{1,t}; \theta_1), F_2(r_{2,t}; \theta_2); \rho) \tag{4}$$

where $F_i(r_{i,t}; \theta_i)$, i =1,2, is the marginal cumulative distribution function of $r_{i,t}$ and θ_i and ρ are the parameters sets of $F_i(r_{i,t}; \theta_i)$ and C, respectively.

Assuming that all cumulative distribution functions are differentiable, the bivariate joint density is then given by

$$f(r_{1,t}, r_{2,t}; \theta_1, \theta_2, \rho) = c(u_{1,t}, u_{2,t}; \rho) \prod_{i=1}^{2} f_i(r_{i,t}; \theta_i) \tag{5}$$

where $f(r_{1,t}, r_{2,t}; \theta_1, \theta_2, \rho) = \partial F^2(r_{1,t}, r_{2,t}; \theta_1, \theta_2, \rho) / \partial r_{1,t} \partial r_{2,t}$; $u_{i,t}$ is the "probability integral transforms" of $r_{i,t}$ based on $F_i(r_{i,t}; \theta_i)$; $c(u_{1,t}, u_{2,t}; \rho) = \partial C^2(u_{1,t}, u_{2,t}; \rho) / \partial u_{1,t} \partial u_{2,t}$ is the copula density function1; $f_i(r_{i,t}; \theta_i)$ is the marginal density of $r_{i,t}$. Thus, the bivariate joint density of $r_{1,t}$ and $r_{2,t}$ is the product of the copula density and two marginal densities.

The marginal models for both returns are given as follows:

$$r_{i,t} = a_{i,0} + a_{i,1} r_{i,t-1} + \eta_{i,t} \tag{6}$$

$$h_{i,t} = c_i + m_i \hat{\sigma}_{i,t}^2 \tag{7}$$

$$r_{i,t} \mid I_{t-1} \sim f_i(r_{i,t}; \theta_i) \tag{8}$$

where $f_i(r_{i,t}; \theta_i)$ is a density function with conditional mean $a_{i,0} + a_{i,1} r_{i,t-1}$ and conditional variance $h_{i,t}$; $\theta_i = [a_{i,0}, a_{i,1}, c_i, m_i]$. Though $\hat{\sigma}_{i,t}^2$, estimated by maximizing (3), captures the patterns for volatility, it could not be necessarily accurate in size and scale. Therefore, we use (7) to linearly correct the bias and scale of $\hat{\sigma}_{i,t}^2$. All parameters $\theta = [\theta_1, \theta_2, \rho]$ can be obtained by maximizing

$$L_C(\theta) = \sum_{t=1}^{T} \log f(r_{1,t}, r_{2,t}; \theta) \tag{9}$$

Once all estimates are obtained, the one-step-ahead forecast for conditional variance of model k is defined as

$$h_{i,t+1}^k = \hat{c}_i^k + \hat{m}_i^k \left(\hat{\sigma}_{i,t+1}^k\right)^2 \tag{10}$$

and then the one-step ahead MVHR is forecasted by

[1] This paper employs four types of copula function commonly used in literature: Clayton, Clayton survival, Gumbel and Gumbel survival. Details please see Nelsen (1999).

$$MVHR_{t+1}^{k} = \hat{\rho}\sqrt{\frac{h_{2,t+1}^{k}}{h_{1,t+1}^{k}}} \quad (11)$$

The returns of a hedged portfolio is given by

$$R_{p,t+1}^{k} = r_{1,t+1} - MVHR_{t+1}^{k} r_{2,t+1} \quad (12)$$

The variance of the hedged portfolio can be characterized as $Var_p^k = Var(R_{p,t+1}^k)$. The hedging effectiveness of the cMEM is evaluated on the percentage variance reduction of the hedged portfolio relative to the OLS static hedging model, and the relative hedge performance is defined as

$$HPI^{k} = -\frac{Var_p^k - Var_p^{OLS}}{Var_p^{OLS}} \times 100\% \quad (13)$$

3. Empirical Results

The daily data of spot and futures for stock price index of Taiwan over 2005/8/26-2010/12/31 are obtained from Taiwan Futures Exchange (TAIFEX). The futures data are nearby contracts. The returns are calculated by

$$r_{i,t} = \left[\log(C_{i,t} / C_{i,t-1})\right] \times 100 \quad (14)$$

where $C_{i,t}$ is the close price at t. The observed volatility is calculated by

$$V_{i,t} = \left[\log(H_{i,t} / L_{i,t})\right] \times 100 \quad (15)$$

where $H_{i,t}$ and $L_{i,t}$ are the highest and the lowest prices, respectively. The daytime return $r_{i,t-1}^{OC}$ is calculated by

$$r_{i,t}^{OC} = \left[\log(C_{i,t} / O_{i,t})\right] \times 100 \quad (16)$$

where $O_{i,t}$ is the open price. Since we have already learned that the sub-prime crisis begins from September 2007, we divide the sample observations into in- and out-of-sample periods before and after the 2007/9/1, respectively. By doing so, we can see if cMEM model could work from the beginning date of the crisis and deliver better performance over the crisis period. To investigate the impact of estimation window on the hedging effectiveness, the window size can be 500, 750, 1000, 1250 or 1500 daily observations.

Table 1 shows the descriptive statistics of spot and futures for the sample periods. The mean returns for spot and futures returns are similar and close to zero, while the daytime return of futures are negative and almost 16 times larger than those of spot. The standard deviations of spot and futures returns are larger than those of daytime returns and similar to the mean of ranges.

Table 2 shows the out-of-sample hedging performance. The results show that Clayton survival copula provides the best performance with 500, 750, 1000 and 1250 estimation window, resulting in more than 10% higher performance against the OLS. With 1500 observations in the estimation window, Gumbel copula gives the best performance. The best copula model, Clayton survival with window size 1250, outperforms the best OLS strategy by 11.425%. Copula models with patterns of upward co-movement, Clayton survival and

Gumbel, provide generally higher hedging performance than do those with patterns of downward co-movement.

4. Conclusion

This paper combines a copula function and multiplicative error models to capture the dependence structure and the volatility patterns simultaneously, named cMEM. We examine hedging performance of the presenting cMEM with different estimation window sizes for the futures contract of Taiwan stock price index. The results have shown that the cMEM with 1250-day window size for Clayton survival, Gumbel and OLS has better performance in which Clayton survival survives during the crisis and has the best out-of-sample hedging effectiveness. The empirical evidence indicates that the cMEM performs well for the turmoil periods.

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Table 1 Descriptive statistics

	$r_{1,t}$	$r_{2,t}$	$r_{1,t}^{OC}$	$r_{2,t}^{OC}$	$V_{1,t}$	$V_{2,t}$
Mean	0.029091	0.028825	0.009501	-0.135799	1.693813	1.405910
Median	0.075047	0.068988	0.045348	-0.111027	1.389729	1.193226
Maximum	6.765088	6.524620	9.598232	5.956979	12.44374	7.402872
Minimum	-8.848390	-6.912347	-7.274301	-5.591600	0.100000	0.146149
Std. Dev.	1.636503	1.413509	1.281418	1.116574	1.103603	0.830577
Skewness	-0.327263	-0.287744	-0.008919	-0.047250	2.219883	1.832962
Kurtosis	6.748854	5.697678	7.992842	5.794572	12.16457	8.234957
Jarque-Bera	1258.151	661.0017	2165.689	679.2374	9009.014	3548.302
Observations	2085	2085	2085	2085	2085	2085

Table 2 Out-of-sample performance

	Clayton	Clayton survival	Gumbel	Gumbel survival	ECM	OLS
Panel A. Variance of hedged portfolio						
500	0.224534	0.215385	0.216331	0.223128	0.242514	0.241709
750	0.221595	0.214790	0.215390	0.218888	0.242075	0.241974
1000	0.220700	0.214761	0.214922	0.216748	0.242501	0.242235
1250	0.224533	0.213929	0.214246	0.217643	0.242235	0.241522
1500	0.226678	0.218343	0.217058	0.219116	0.244806	0.243586
Best	0.220700	0.213929	0.214246	0.216748	0.242075	0.241522
	(1000)	(1250)	(1250)	(1000)	(750)	(1250)
Panel B. HPI						
500	7.106	10.891	10.499	7.687	-0.333	
750	8.422	11.234	10.986	9.541	-0.042	
1000	8.890	11.342	11.275	10.522	-0.110	
1250	7.034	11.425	11.293	9.887	-0.295	
1500	6.941	10.363	10.891	10.046	-0.501	
Best	8.621	11.425	11.293	10.257	-0.229	

The number in parenthesis is the size of estimation window for the best model. “Best” denotes the best model across different window size.

以 Copula Multiplicative Error Model 建構股票期貨動態避險策略

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摘要

本研究結合 copula 函數與 Brownlees and Gallo (2009)的單變數 multiplicative error model(MEM)架構，建構台灣股票指數期、現貨動態避險模型(copula MEM, cMEM) ，估計最小變異最適避險比率 (minimum-variance optimal hedge ratio)，進而比較各模型在不同估計視窗下的避險績效。本文實證結果發現，在估計視窗 1250 筆時，Clayton survival、Gumbel 及 OLS 都有較佳的避險績效，其中以 Clayton survival 在金融危機期間擁有最佳的避險績效，較最佳的 OLS 模型改善超過 11%。本文結果證實 cMEM 模型在危機期間具有不錯的避險表現。

關鍵字: 避險比率; copula;相乘誤差模型; 相關結構;股票期貨